

# The Focusing on Compression to Thinkable Concept as Shift from Procedures to Concept: Evidence from Thai Classroom

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## Abstract

The distinction between procedural and conceptual learning has long been a topic of discussion in mathematics education and the idea of compression into thinkable concepts that enable the individual make links between them (Tall, 2007). In addition to, the compression to thinkable concept was to be thinking mechanism arising naturally and important thinking tool in developing concepts (Gray & Tall, 2002). The major act of compression to thinkable concept shifts attention from the sharing procedure as a sequence of steps to the effect of that procedure (Gray & Tall, 2007). So, the compression to thinkable concept is one important way in order to understand in development from procedures to concept.

This article presented example the compression procedures in problem solving to thinkable concept under context using Lesson Study and Open Approach, with the four of first graders a project school supported by CRME. In this study based on the framework that purposed by Tall and Isoda (2007) and Poynter (2004).

Under context using Lesson Study and Open Approach provide to students manipulated with designed materials for supporting and checking their various symbolic thinking before into same effect through parallel compression 5 steps into thinkable concept in blending embodiment and symbolism. In embodiment from step-by-step action, to the possible choice of several different action, to seen the overall effect as same effect that can be carried out in various ways, to compressing it as effect as prototype, to awareness and appreciation of effect used later. In symbolism from step-by-step procedure, to the possible choice of several different procedures, to seen the overall effect as a general process that can be carried out in various ways, to compressing it as thinkable concept, to awareness and appreciation of thinkable concept in extending mathematical structure. Consequently, the students used considering the way of thinking and refined the ideas in order to develop a major idea from different procedures in problem solving.

**Keywords:** lesson study, open approach, compression to thinkable concept, precept

## 1. Introduction

The distinction between procedural and conceptual learning has long been a topic of discussion in mathematics education and the idea of compression into thinkable concepts that enable the individual make links between them (Tall, 2007). In addition to, the compression to thinkable concept was to be thinking mechanism arising naturally and important thinking tool in developing concepts (Gray & Tall, 2002). The major act of compression to thinkable concept shifts attention from the sharing procedure as a sequence of steps to the effect of that procedure (Gray & Tall, 2007).

In addition, Gray and Tall (2007) suggested that the mechanism of compression is the result of refining the concept using different procedures to solve problems, to create the concept, implementation of such a man can think in general, and mathematical thinking. In particular, the level of complexity rises steadily and this is the main to the development potential of thinking more. Tall (2007) discussed the new method significantly related to teachers and researchers will need to consider such concepts. Emphasized by the current mathematics curriculum should focus on the concept of how to solve the problem. Which is essential for the implementation of the compression into thinkable concept. Even so, there is no evidence to show these things clearly, so

researchers and educators need to understand these things and show these things to see out to be more concrete, to create understanding for the teachers, students, parents and all those involved in further education (Tall, 2007).

The ideas consistent and important ideas to consider compression to thinkable concept of students. Idea is consistent with the way in finding answers or results obtained by considering what dually of symbols in the calculation is the process of using different procedures to solve problems and conceptualize that created apply the refined concept of calculating symbolic (Gray & Tall, 1994) and Tall (2004) suggest that as of now, the change of the process to the concept can not be built obvious, or can not be observed easily. So the idea that seen as a substantial and consistent is the concept of Tall and Isoda (2007) describes the classes that implement Lesson Study caused students an idea or concept of the stages through implementation compression to thinkable concept of the 4 sequences of procedures used in the solution to the outcome. Such an idea is based on the idea of Tall (2006) discusses the compression into thinkable concept through the implementation of the 5 sequence steps. (As will be discussed later).

In Thailand-based classroom apply Lesson Study and Open Approach by integrating together. As a form of teaching that is specific to develop the mathematical thinking of students. Including driving teaches classes for analysis and creating the concept of Inprasitha et al. (2007) suggest that Lesson Study aims to develop and improve lessons in the context of a real classroom. Focus on the changes that need to happen to improve the learning of students is divided into 3 basic steps including to create a lesson plan together, the classroom observation, and sharing reflecting the lessons together including the creation of problem situation, the material design students to take action to support critical thinking and problem solving of students and focus on the sequence of instruction based on the concept of Inprasitha (2010) Open Approach is that the teaching methods to operation activities in the classroom consists with 4 stages: Posing open-ended problem situation, Student's self-learning, Whole class discussion and comparison, and Summary through connection. The teaching are driven to solve problems on their own and led to the understanding that the essence of the solution.

Also this reason this articles will offers the compression to thinkable concept from procedures in problem solving to conceptualize classroom-based Open Approach on considerations. Empirical evidence, through a theoretical framework. The analysis methods that students use to solve problems, to create a concept of such studies are more tangible.

## 2. Context of Education

### 2.1 Lesson Study

Lesson Study are innovative use of analysis classes and developmental mathematics students in Thailand, Inprasitha et al. (2007) have introduced the concept of Lesson Study from Japan used by adjusted to suit the conditions, collaboration of stakeholders in the education classes consist of 3 phases including: the creating of teaching plans together, the observation of teaching together and to reflect the lessons together. Each step is described in brief as below:

- 1) The creating of teaching plans together: the steps that the team plans to build a teaching experience of the teachers. As the ideas and experience of past students. Consider the sequence of instruction. Situations and problems that are used to create the plan. Including the idea of student expectations and analysis textbook of Japan (GAKKOH TOSHO. Study with Your Friends MATHEMATICS for Elementary School 1st grade (Gakkotosho, 2005), which through research and development of specialized and sequence covering process and mathematical thinking of students Grade 1 applied to create events and situations. Moreover, the design of scenarios to illustrate how students think and plan in order to teach the Open Approach.
- 2) The observation of teaching together. A step further in creating lesson plans together. By lesson plans that have been created to teach by teaching with Open Approach. (As will be discussed later) At this stage, the team plans to observe teaching. Students happened how students use to solve problems. Student interaction with the material designed of Students exotic. Students are expected their idea. Students are not expected their idea. The record of what was observed and apply instruments to collect data.
- 3) To reflect the lessons together. A step team created lesson plans and teaching reflect together facets. The ideas of students taking the class from observing classes together.

### 2.2 Open Approach

Open Approach as a sequence of instruction that has been designed from the stage to create a lesson plan together of the Lesson Study process. To enable students to learn the capabilities of the students and the ideas of

the solution and a tool for change the new class. Taught by Open Approach aims to enable students to learn mathematics by the students according to the idea of Inprasitha (2010) suggest that in class sequence taught by means of Open Approach integrated with Lesson Study and plan accordingly with open approach consists of four stages.

- 1) Posing open-ended problem situation: In this stage, teachers who present problems. To students and give students a problem to solve manually using scenarios. The use of material designed for students to attend to a solution.
- 2) Student's self learning: In this stage, students start to do for solve the problem manually including the use of various methods. To think, to find answers and create ideas that solve a goal. Students interact concrete material and material designed to think in symbolic computation and leads to the concept of themselves.
- 3) Whole class discussion and comparison: At this stage, the students presented their ideas to the class to discuss the class together to learn from each other and gives students the opportunity to explain the ideas of self, once again, the students examine their own ideas happen.
- 4) Summary through connection: At this stage, the teacher is the conclusion. Lessons in class and links the ideas of students going on. Allows students to check ideas that occur. To teaching students how to open push to solve problems on their own and contribute to understanding the essence of the solution. To prepare the students to learn properly. This article show in area of sequence taught by the open approach to analyze the mechanism of abstraction of the students in order for students to achieve the self-learning of how students use to solve a variety of promotional ideas. In sum, happening and encouraging students to important concepts from the solution.

### **3. Framework Concept**

The concept is the purpose of the lesson and that is what teacher want to happen to all students. By seeking to appropriate learning experiences for students. In the context of classes that use the Open Approach, which was driven to the teaching and learning using Lesson Study has used the situation problem open-ended driven students to practice with material designed to support or check for a symbolic computation to solve the problem of students' abilities and interests of each student independently. Using various methods in solutions help promote and develop students to create concepts by themselves, so the analysis procedures to solve the problem to a notion that occurs in the classroom using innovative Lesson Study and Open Approach, to open framework concept as follows:

#### *3.1 Procept*

Procept as the words that Gray&Tall (1994) coined by a naturally dually of the process and concepts of the same symbols to represent the process such as in addition and subtraction and productivity of the process, that is. Sum and difference and the combination of 3 components as: process to make the calculations in solving problems, to find solutions for answer. Mathematical concepts and symbol for use in the calculation is presented as a process or concept and make the result (effect) of the same by the same concept. Using the idea of the concept is the natural process of refining the concept developed into a higher order. From the process and led to the concept. But the action also can not be considered a tangible, to consistent with the concept of Tall (2004) discusses the current changes to the process of how the concept can not be seen as factual. So it should be studied and explained the concept on the base of empirical evidence consistent.

#### *3.2 Compression to Thinkable Concept*

Tall&Isoda (2007) describes the classes that implement Lesson Study does not limit the student's way of thinking has resulted in a wide variety of students. Such concepts consider different ways to solve problems and make important concepts that occur through four stages Continuation various methods to solve the problem to the results as follows: (1) using the same method to the successful implementation of the solution (a procedure) (2) to use different methods for different customers. In order to choose the most effective. (Multi-procedure) (3) to realize how many different sequences may involve different steps. But how different is different to the same results (an overall process) (4), the results are considered as the concept itself (a thinkable concept) (or a conceptual process (procept) based on the idea of Gray & Tall (1994) consider symbol the process as a bis-manner of operation of the various methods used to solve the problem. The notion that results from the action of using different methods such concept is based on the concept of Tall (2006) consider the operations refining concepts through 5 steps higher ongoing: There are, (1) pre-procedure, (2) procedure (3) multi-procedure (4) process, and (5) the understanding of the process is concept.

In addition, the concept of Poynter (2004) discusses how to implement the concept into a refined concept. By considering order of operations, refining the concept of interaction with objects physically as follows, (1) acts as a physical (action) (2) the consequences of the actions are the same (same effect) and (3) the consequences of the action is the same format (effect as prototype) based on the concept of the Tall (2007) discussed the implementation of the parallel worlds of physical and interaction with the media or object physically caused in a manner parallel to the symbolic figure of the refined concept into a concept.

#### 4. Methodology

Used Teaching Experiment to see the students' concept formation from as shift procedures to concept in compression to thinkable concept, observed several ways they use to solve problem and chose important concept to build thinkable concept. Researcher treated Open Approach as a sequence of teaching to provide various the natural thinking way in class to study target group using video, photographs, tape recording, field notes, interviewing teachers, teacher trainees and collaboratively observed in class to analyze the data as in framework. Research embedded to study learning and teaching culture for 3 years, target group was one of four school in the project under Center for Research in Mathematics Education, Faculty of Education, Khon Kaen University for 5 years. It was a small school with only one class in each grade. The first grade students were used Lesson Study and Open Approach in 3 steps collecting data as following:

Teaching plans were divided into two periods: before semester and after semester. Before semester, teachers, observers, internship mathematics student teachers, research team wrote teaching plan in units and periods, learning activities, objectives and open-ended problems using Japanese textbook. It was team collaboration of 4 schools. During semester, there were teaching plans on Tuesdays for this school, using students' concept in class students' background knowledge, experiences as well as expecting students' ideas in doing mathematical activities, open-ended problems. There was instruction for students to reveal thinking concept during doing mathematical activity and to create teaching plans and materials together. In class teaching focused on 4 steps of Open Approach: posing open-ended problem situation, student's self learning, whole class discussion and comparison, and summary through connection. The data was collected by tape recording and analyzed with the other steps.

At the teaching step, teachers taught in class after team planning, focused on the important of thinking time, and the students present their work in front of class. Teachers walked around to see the students' concept, to arouse them showing their way of thinking, and help them in class presentation by using authentic teaching materials. Observer team (teachers, internship mathematics student teachers, school coordinators, and researcher) participated at this step in class by observing students' ideas and oral presentation in the classroom. Observer teachers, teachers, internship mathematics student teachers, research team, school administrators participated at this step. They observed students' task: oral and action to build thinkable concept according to four steps of Open Approach. Observer teachers, teachers, internship mathematics student teachers, research team, school administrator and experts participated at the reflecting step in each classroom from grade 1 to grade 9. They observed students' concept and their tasks. The data was collected by tape recording, video, and analyzed.

At the collaboration observation step, collected data from teaching experiment in class to see as procedures to concept through the compression to thinkable concept with conceptual analysis, using video recordings, field notes, pictures, interviewing witness in instruction background assembles (teachers, observer teachers and internship mathematics student, teachers) and analyzing students' tasks with triangulation. The data was from class observing, interviewing and students' task. Exemplar analyzing students' concept using problem situation "Chicken with chick (9+7)" from team collaboration to build and analyze classroom teaching from planning lesson focused an open-ended problem situation. Observed students' oral and action presentation and analyzed their tasks. Analyzed empirical evidence in teaching scenes to understand how the students' concept formation as shift from procedures into concept through the compression to thinkable concept. On concept of substraction for comparing different number.



Figure 1. Cycle of lesson study including 3 phases

## 5. Example for Analysis

For example, to analyze how to solve problems, to develop into the concept of students in classes that use Lesson Study and Open Approach is divided into 3 steps.

- 1) Overall instruction in the use of traditional open approach (overview classroom by Open Approach)
- 2) Analysis of how to think, solve problems of students (The procedures of students' thinking in problem solving).
- 3) Analysis of the refined concept classes that use an Open Approach to solution to conceptualize (The step of compression to thinkable concept in classroom using Lesson Study and Open Approach from procedures in problem solving into concept)

Before beginning to analyze this example, this study analyzed the 2 previous problem situations and found that students found the concepts they have learned to use as a tool in problem solving and is the basic knowledge before this class period in terms of number and addition. The analysis of the two-problem situation can be summarized as follows;

- a) The problem situation on “Slacken? a ping pong ball into the box” (5 numbers) to prepare for the students' readiness and understanding in terms of numbers and tools used in addition and subtraction.



**Results:** The students classified the 5 numbers into six types: (0,5), (1,4), (2,3), (3,2), (4,1), (5,0)

- b) The problem situation in “Sandbox toys and sports” (9+4) for students to perform the addition and understand the meaning of combined addition by using the concept of decomposition and creating the numbers by using the 10 as a tool in finding the answers.



**Results:** Students performed the addition by using the decomposition of the front or the back number in order to make it as 10 and add the 10 with the remaining amount to find out the results.

And this situation:

c) The problem situation in “Chicken with chick” ( $9 + 7$ ), for students to perform the addition and understand the meaning of an increasing addition approach by using the concept of decomposition and creating the numbers by using the 10 as a tool in finding the answers.

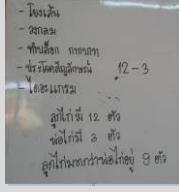


**Results:** Students performed the addition by using the decomposition of the front or the back number in order to make 10 and add the 10 with the remaining amount in finding the results.

### 5.1 Overview Classroom by Open Approach

Chart 1. Overview traditional teaching methods to open the minds of students to consider the concept of students as follows

Teaching Approach	Teacher instruction	Student Learning
	teacher/evidence/	teacher/evidence/
1. Posing open-ended problem	 <p>Teacher caught his problem situations chicken with chicks and take on board the focus on the questions to show students how to think. Number of Ports chicken or chicks which one over and over each other much.</p>	 <p>Students think as the instruction, the teacher pointed out that the number is well over and over it. Students answered “Chicken over 8 chicks” students interested in the problem situation and trying to find answers to other methods.</p>
2. (Student's self learning)		

	<p>Teachers walk and provide suggestion feedback when students ask questions. Allows students to gain confidence in their own thinking more and teachers encourage students to show you how to think like variety.</p>	<p>Students try to solve the problem and find the answer to the problem. By doing things as a couple and the concept of friends in groups to brainstorm. There are a variety of students how to think. Both the paired associate By writing the number in the tens of chicks and how many points each chicken. Drawing block instead of chicks and chickens his father. Using diagrams to split them into 10 and led 10 to remove the negative to find the answer.</p>
<p><b>3. Whole class discussion</b></p>	 <p>Students presented their ideas to the class and explain to friends, listen to learn how to exchange ideas with each other.</p>	 <p>Students explain their thinking through and a friend were asked to share their thinking. Enabling students to explain the concept of self again from showing how to remove the use of 10 as a tool to find answers</p>
<p><b>4. Summary through connection</b></p>	 <p>Teacher write all of students a brief idea that has happened, like a circle linking the father hen with chicks a cross and a symbol</p>	 <p>Students tell all the methods used to find answers to the teacher writes on the board the way of thinking and the answer is in finding the answer.</p>

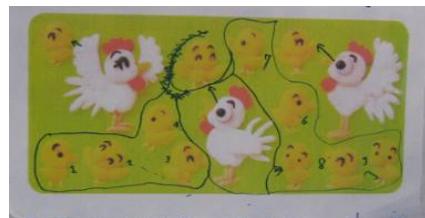
### 5.2 The Procedures of Students' Thinking in Problem Solving

1) Students use the circle chicks, directed to count and write numbers to 12 and all the chicks.

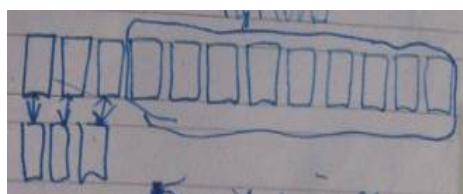
And write number of 1-3 show a number of chicken.



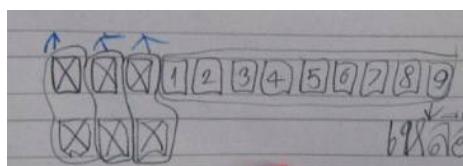
2) Student's paired the chicken with chicks and circle the remaining chicks and write the number in the chicks are left with 9.



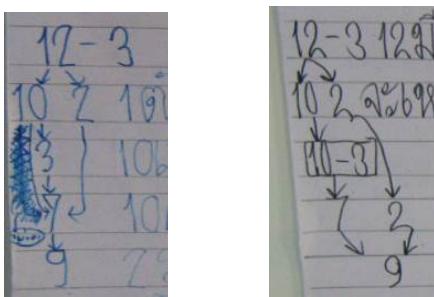
3) Students used to paint and draw blocks instead of chicks 12 blogs, and draw blog instead of three chickens then tied to match a student's blog chicken with chicks, remaining blocks that do not match 9 blocks.



4) Student draws 12 blogs instead chicks and sketch blog instead of 3 blocks of chicken instead of chickens, and cross matching his father-chicken and write the blog for the rest of the match for 1-9.



5) Students take a total of 10 then led number 10 to make it a negative number to be deleted. Which in this case is 3, then be integrated with the rest of the separation of 10 the answer is 9



When considering how to think of the students. Student writes numbers are to be counted. Draw blocks to represent the number of chicken with the number of chicks available and pull the block out to show to pull out of a number of existing by cross blocks drawn out to 3 of the existing blocks 12 blocks and the remaining blocks are 9 blocks and using other methods, such as cross-matching lines. Using diagrams to 10 out of 12 then 10 minus 3 answers 7 and the number with 2, answer came to b 9 corresponds to the observation data into a writing lesson plans. Teachers emphasize to students the knowledge and concepts learned by using the concept of 10 to be used in the negative. Using diagrams as a tool to extract the number and total number of 10 as a tool to solve problems.

### 5.3 The Step of Compression to Thinkable Concept in Classroom Using Lesson Study and Open Approach from Procedures in Problem Solving into Concept

At the stage of the students to learn on their own, show students how to think like variety. Which are driven by the image-driven scenarios for students to show how the students' ideas or key concepts from various methods, used in the solution and action to create the concept of students from various methods. That students use to solve problems. At this stage of analysis based on the concept of Tall and Isoda (2007) and Poynter (2004) discusses the process of refining the concept into a concept that can think for themselves in the context of education

classes and methods open. Which does not restrict students to use a single method to solve the problem, but beneficial to students using a variety of methods to solve problems and push to the concept or notion of students to solve problems. The analysis is supported by the concept of student interactions with the problems such as the circular, cross-matching purposes, the match and implementation of a symbolic figure for the calculation is based on the following hierarchy:

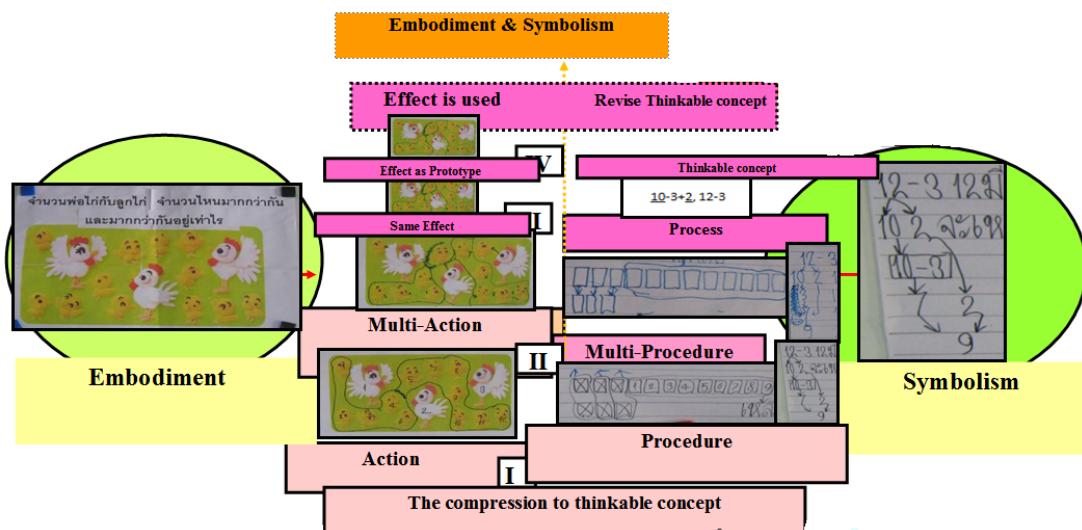


Figure 2. The compression to thinkable concept as shift from procedures to concept

The results correspond with the teacher's reflect said that the students apply their knowledge to create a number and separate number and use 10 as a tool in a problem solving. The concept of the students is a diagram of negative result of  $10+2-3=12-3$ , which is the same answer 9 and students using material design as a tool to monitor the situation concept of the problems. Student considered procedure is a simple procedure to calculate which is supported by the use of the design material. To make 10 has an important role to support and check symbolic idea. Also it serves to validate the concepts and gives students the confidence to answer students' thinking. And it provides a tool for students to think of the idea by making a total and subtraction, which is a simple and efficiency idea.

When considering the implementation of a refined concept of the physical and symbolic world is working in parallel with the development of methods for the solution to conceptualize. Consistent with the concept of Gray and Tall (2007) have suggested the important role of the refining concept to the attention of the methods used together, which is a sequence of steps. The solution to conceptualize. Classroom-based education classes and open approach to prepare students to perform math activity ideas with action to the media to support and monitor the student a variety of symbolic thought before to the concept In this area are helpful in determining the refined concept 5 steps parallel world of the physical and symbolic through the development of a method of solution to conceptualize, are summarized below.

- 1) The physical world: The students making with the media in a way, step by step. To the selection of the best action from the action of different media. To see the overall effect is the same model can be implemented in various ways. To refined the concept to the implementation of the action into the same pattern. To the recognition and value of the format happens to be continuing to use it.
- 2) In the symbolic: Students use the solution step by step. How to select the best of a variety of different ways to solve problems. To see different ways, used to solve the common processes of various methods. To continue refining the concept of how to solve the problem to become a concept and to recognize and appreciate the concepts that arise in the extension of mathematical structure.

## 6. Conclusion

Under the context of educational classes and open approach to prepare students to interact with the media to support the idea and be sure to think symbolically and the space to consideration in its 5 successive stages of the refining concept of building concept from procedure to concept. The combination of the physical world and the

world as a symbolic sequence of steps in parallel. In embodiment from step-by-step action, to the possible choice of several different action, to seen the overall effect as same effect that can be carried out in various ways, to compressing it as effect as prototype, to awareness and appreciation of effect used later. In symbolism from step-by-step procedure, to the possible choice of several different procedures, to seen the overall effect as a general process that can be carried out in various ways, to compressing it as thinkable concept, to awareness and appreciation of thinkable concept in extending mathematical structure. Consequently, the students used considering the way of thinking and refined the ideas in order to develop a major idea from different procedures in problem solving.

Approach to develop students' thinking to the process of creating the concept and the concept of the end. From one of the various methods and processes to solve problems. An important concept in the solution and crystallization occurred to bring the concept to the next solution. Such methods occur in both the physical and symbolic world. Refining the concept of how to solve the problem to the concept of linking procedure and concepts perfectly. However, this study should expand study and analyzed in mathematical activity that are consistent with the mathematical problem situation in students' daily life and linking between real world and mathematical world through comprehension to thinkable concept from procedures to concept. The study is limited by the group were students grade 1, the study was not able to see the long-term development of learning should be studied in sequence of Grade 2 and 3.

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